Network Approach versus State-space Approach for Strapdown Inertial Kinematic Gravimetry

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Abstract. The extraction of gravity anomalies from airborne strapdown INS gravimetry has been mainly based on state-space approach (SSA), which has many advantages but displays a serious disadvantage, namely, its very limited capacity to handle space correlations (like the rigorous treatment of cross-over points). This paper examines an alternative through the well known geodetic approach, where the INS differential mechanization equations are interpreted as a least-squares network parameter estimation problem. The authors believe that the above approach has some potential advantages that are worth exploring. Mainly, that modelling of the Earth gravity field can be more rigorous than with SSA and that external observation equations can be better exploited.

Keywords. INS/GPS, airborne gravimetry, kinematic gravimetry, geoid determination, INS calibration, network approach (NA), state-space approach (SSA).

1 Motivation

A relatively recent technique in the field of airborne kinematic gravimetry is the combined use of strapdown inertial navigation systems (strapdown INS or SINS) —or inertial measurement units (IMU)— and the Global Positioning System (GPS) —Schwarz (1985). We will refer to it as INS/GPS-gravimetry. INS/GPS-gravimetry uses the differences between the linear accelerations measured by the accelerometers of an IMU and the accelerations derived from GPS. INS/GPS-gravimetry is mainly affected by two error sources: short term GPS-derived acceleration errors and long term INS inertial sensor errors —Schwarz and Li (1995).

For geoid determination applications, short term errors —i.e., the noise of GPS-derived accelerations— have been identified as one of the limiting factors of the technique. Fortunately, the situation is likely to improve significantly with the advent of the European global navigation satellite system Galileo because of its higher signal-to-noise ratio and with the subsequent use of hybrid Galileo/GPS receivers.

On the long wavelength side of the problem, the correct measurement of gravity —or, rather, of the anomalous gravity field— with INS/GPS-gravimetry depends on the correct separation of the INS/GPS errors from the actual variations of the gravity field itself (now, the long-wavelength bias stability is the limiting factor). This separation is, in principle, feasible because of the different characteristics of the two signals: errors of the inertial sensors can be reasonably modeled as time functions, whereas the variations of the gravity field are, strictly, spatial functions. (Understandably, so far, most of the research has focused on the INS/GPS short wavelength errors as the practical use of the technique and its competitiveness with traditional terrestrial gravimetry is bounded by, moderate to high, precision and resolution thresholds.) An improvement of the calibration of inertial sensors may be seen as an improvement of the long wavelength errors of INS/GPS-gravimetry. By doing so, we are not only achieving an overall improvement of INS/GPS-gravimetry but, in particular, we are extending its spectral window of applicability. This extension might be instrumental to the integrated use of GOCE gravimetry and INS/GPS-gravimetry as the sole means of gravimetry for geoid determination.

In this paper, we investigate algorithms to better calibrate the systematic errors of the inertial sensors. More specifically, we investigate an alternate procedure to the traditional Kalman filtering and smoothing. The advantage of the "new" procedure is that it can assimilate all the information available in a gravimetric aerial mission; from ground gravity control to the cross-over conditions, among other observational information types. The proposed procedure is nothing else than *geodesy as usual* in that we redefine the INS/GPS-gravimetry problem as a network adjustment problem —early studies can be seen at Forsberg (1986) for least-squares methods in land-based and helicopter-based inertial gravimetry.

Last, we note that better and more reliable algorithms for inertial sensor calibration may allow the use of low noise inertial sensors even if they suffer from large drifts. This, in turn, has a positive impact on the low frequency end of the INS/GPS-gravimetry spectral window.

2 INS/GPS-gravimetry: geodesy as usual

So far, extraction of gravity anomalies from INS/GPS-gravimetry has been mainly based on a state-space approach (SSA): the output of the stochastic dynamical system defined by the INS mechanization equations is Kalman-filtered and -smoothed with the GPS-derived positions and/or velocities – see Schwarz (1985), Wei and Schwarz (1990), Schwarz and Li (1995), Tomé (2002).

In INS/GPS-gravimetry, the separation of the INS/GPS errors from the variations of the gravity field is obtained by the use of appropriate models —e.g., stochastic differential equations— for the IMU sensor systematic errors and for the gravity field anomalies. Given the INS mechanization equations, the IMU calibration equations and the gravity field variation equations (sic), the SSA generates "optimal" estimates for the IMU trajectory (position, velocity and attitude), for the IMU errors and for the gravity field differences with respect to some reference gravity model.

In INS/GPS-gravimetry, the SSA is essentially given —Wei and Schwarz (1990)— by

$$\begin{aligned} \dot{r}^e &= v^e \\ \dot{v}^e &= R_b^e(f^b + w_f^b) - 2[\omega_{ie}^e \times]v^e + g^e \\ \dot{R}_b^e &= R_b^e\,[(\omega_{ib}^b + w_\omega^b)\times] - [\omega_{ie}^e \times]R_b^e \end{aligned} \qquad (1)$$

where r^e and v^e are the position and velocity vectors in the Conventional Terrestrial frame (e); R_b^e is the transformation matrix from the body frame (b) to the e-frame; $\omega_{ie}^e = (0, 0, \omega_e)^T$ where ω_e is the rate of Earth rotation; g^e is the gravity vector as a function of r^e; w_f^b and w_ω^b are the generalized white-noise processes of the specific force (f^b) and angular velocities (ω_{ib}^b), inertial observations respectively.

The numerical solution of this system can take many different forms which may be model-based or not, see Hammada and Schwarz (1997). It should be noted that hardly any of the active groups working on these problems uses Kalman filtering as a standard procedure today. Typically a two-step procedure is employed: in a first stage, FIR filtering or something similar to take care of time-dependent errors, and in a second stage, a cross-over adjustment to take care of the spatial structure of the gravity field.

The key to overcome SSA limitations is to look at the system equations (1) as a stochastic differential equations (SDE) that, through discretization, leads us to a time dependent geodetic network as discussed in —Térmens and Colomina (2003), Colomina and Blázquez (2004)— for geodetic, photogrammetric and remote sensing applications. A time dependent network is a network such that some of its parameters are time dependent or, in other words, stochastic processes. A time dependent network can be seen as a classical network that incorporates stochastic processes and dynamic models. A classical network can be seen as a particular case of a time dependent network.

To solve a time dependent network is to perform an optimal estimation of its parameters which may include some stochastic processes. As usual, the solution of the network will end in a large, single adjustment step where all parameters, time dependent and independent, will be simultaneously estimated.

In a time dependent network we may have static and dynamic observation models. A static observation model is a traditional observation equation. A dynamic observation model —or a stochastic dynamic model— is an equation of the type

$$f(t, \ell(t) + w(t), x(t), \dot{x}(t)) = 0 \qquad (2)$$

where f is the mathematical functional model, t is the time, $\ell(t)$ is the time dependent observation vector, $w(t)$ is a white-noise generalized process vector, $x(t)$ is the network parameter vector and $\dot{x}(t)$ the time derivative of $x(t)$. Note that $x(t)$ contains stochastic processes that, in particular, may be random constants thus including traditional time independent parameters. The discretization of the dynamic observation models together with the static observation models and further network least-squares adjustment will be referred to as the network approach (NA).

In general, NA has many potential advantages compared to SSA: parameters may be related by observations regardless of time; networks can be static and/or dynamic; covariance information can be computed selectively; and variance component estimation can be performed. In the context of INS/GPS-gravimetry the authors believe that some of the NA potential advantages are significant: modeling of the Earth gravity field can be more rigorous than with the SSA; external observational information can be better exploited; and more information for further geoid determination is produced. The main drawback of NA is that it cannot be applied to real-time INS/GPS navigation but this is certainly not an issue for a geodetic gravimetric task.

3 INS/GPS-gravimetry models for the NA

In this section we review the dynamic and static observation models that can be assimilated by the NA for INS/GPS-gravimetry. We note that the set of dynamic observation models corresponds to what is called *the system* in stochastic modeling and estimation. Analogously, the set of static observation models corresponds to what is called *the observations*. In the context of time dependent networks —Colomina and Blázquez (2004)— the names dynamic and static observation models are used to highlight the fact that we build our network from observations that contribute to the estimation of parameters either through dynamic or static equations.

3.1 Dynamic observation models

The dynamic observation models are, essentially, two. One model is the set of the INS mechanization equations and the other model expresses the "continuity" of gravity along the aircraft trajectory.

The mathematical model associated to SINS navigation is given by the well-known mechanization equations (1), that are usually extended with the angular rate sensors and accelerometers calibration states and models. The choice of these models has to guarantee that the estimated calibration parameters will not absorb other kind of effects, specially anomalous gravity. Investigations published in Nassar et al (2003) show that a linear calibration model is not sufficient, but in this paper, to fix the ideas and for the sake of simplicity we restrict intentionally the calibration states to time dependent biases:

$$\begin{aligned} \dot{r}^e &= v^e \\ \dot{v}^e &= R_b^e(f^b + w_f^b + a^b) - 2[\omega_{ie}^e\times]v^e + g^e \\ \dot{R}_b^e &= R_b^e[(\omega_{ib}^b + w_\omega^b + o^b)\times] - [\omega_{ie}^e\times]R_b^e \\ \dot{o}^b &= F_{gyr}(o^b) \\ \dot{a}^b &= F_{acc}(a^b) \end{aligned} \quad (3)$$

where F_{gyr} and F_{acc} are the calibration model functions of the angular rate sensors (o^b) and accelerometers biases (a^b). (Needless to say, the calibration functions and the calibration states depend on the type of sensors.)

The system (3) can be extended with a new mathematical model —GDT model— that shows the changes of the gravity disturbance along the trajectory of a moving vehicle with respect to time. The changes of the gravity vector g^e along the trajectory with respect to time can be given —Jekeli (2001)and Schwarz and Wei (1995)— by

$$\dot{g}^e = (\bar{G}^e - [\omega_{ie}^e\times][\omega_{ie}^e\times])v^e = \Delta G^e v^e, \quad (4)$$

where $\bar{G}^e$ is the gravitational gradient tensor. For the gravity disturbance vector, similar differential equations — $\dot{\delta g}^e = \Delta G^e v^e$ — are obtained. If no gravity gradiometer measurements are available $\Delta G^e v^e$ can typically be modelled by simple stochastic models. Then, to fix the ideas and to simplify the modeling, the gravity disturbance can be represented by a random walk.

Now the dynamic observation models formed by SINS mechanization equations (3) and GDT model are:

$$\begin{aligned} VEL: &\quad \dot{r}^e = v^e + w_0 \\ FB: &\quad \dot{v}^e = R_b^e(f^b + w_f^b + a^b) - 2[\omega_{ie}^e\times]v^e + \\ &\quad +\delta g^e + \gamma(r^e) \\ WIB: &\quad \dot{R}_b^e = R_b^e[(\omega_{ib}^b + w_\omega^b + o^b)\times] - [\omega_{ie}^e\times]R_b^e \\ OB: &\quad \dot{o}^b = F_{gyr}(o^b + w_o^b) \\ AB: &\quad \dot{a}^b = F_{acc}(a^b + w_a^b) \\ GDT: &\quad \dot{\delta g}^e = w_g^e \end{aligned}$$

These models are time dependent equations of the type (2), where $\ell(t) = (f^b, \omega_{ib}^b)^T$ and $x(t) = (r^e, v^e, R_b^e, \delta g^e, a^b, o^b)^T$.

3.2 Static observation models

The static observation models considered are: the coordinate update point (CUPT), the velocity update point (VUPT), the zero velocity update point (ZUPT), the gravity update point (GUPT), the static gravity update point (SGUPT) and the cross-over points (XOVER).

CUPT model. A coordinate update is a point where the position of the platform is known from an independent procedure (usually GPS). The CUPT equation is $p_0 + w_p = r^e$.

VUPT model. If instead of the position the speed is also known, the associated equation is $v_0 + w_v = v^e$.

XOVER model. Usually, the trajectory of gravimetric flight follows a regular pattern. The intersection points of the trajectory are known as cross-overs. Since, in practice, actual intersections are hard to materialize, horizontal observations with small height differences are allowed for the cross-over points. The cross-over observation equation imposes that gravity is the same in coincident points.

ZUPT model. The zero velocity update is based on $v^e = 0$ and it is widely used in terrestrial inertial surveying. In a gravimetric flight, it can only be applied at the beginning and at the end of the survey. This model is equivalent to a VUPT with $v = 0$.

SGUPT model. For every ZUPT equation, gravity can be considered as a constant function. It can be seen as a XOVER observation.

GUPT model. If gravity is known in some point of the trajectory, the following equation is obtained: $g_0 + w_g = \delta g^e + \gamma(r^e)$.

3.3 Discretization of the dynamic observation equations

The dynamic observations equations are SDE. SDE arise naturally from real-life ODE (ordinary differential equations) whose coefficients are only approximately known because they are measured by instruments or deduced from other data subject to random errors. The initial or boundary conditions may be also known just randomly. In these situations, we would expect that the solution of the problem be a stochastic process.

Like in ODE theory, certain classes of SDE have solutions that can be found analytically using various formulas, and others —most of them— have no analytic solution. There are several numerical techniques to solve SDE —Kloeden and Platen (1999). All of them are based on their correct stochastic discretization which is not a trivial issue.

Now, in this paper, we will limit the discussion to a simplest approximation method: the explicit midpoint method or leap-frog method. Consider for a function $x[n]$, $\dot{x}[n] = \Lambda(x, n) = (x[n+1]-x[n-1])/(2\delta t)$. This method is not generally acceptable, because the existence of weak stabilities. However, in this paper, it suffices to illustrate the use of NA for INS/GPS-gravimetry.

Then, the previous equations —the dynamic and the static— can be transformed into a finite set of observation equations. They can be discretized and afterwards written as $\ell + w = F(x)$, where ℓ are the observations (in our case f^b, ω^b_{ib}), w are the residuals of ℓ and x are the parameters to be determined (r^e, v^e, δg^e, a^b, o^b, q):

$$\begin{array}{ll} VEL: & 0 + w_0 = v^e[n] - \Lambda(r^e, n) \\ FB: & f^b[n] + w^b_f = -a^b[n] - \\ & -R^b_e(q,n)\,\{\delta g^e[n] + \gamma(r^e[n]) - \\ & -2[\omega^e_{ie}\times]\,v^e[n] - \Lambda(v^e, n)\} \\ WIB: & \omega^b_{ib}[n] + w^b_w = -o^b[n] + R^b_e(q,n)\,\omega^e_{ie} + \\ & +2M_q(q,n)^T\,\Lambda(q,n) \\ OB: & 0 + w_0 = \Lambda(o^b, n) - F_{gyr}(o^b[n]) \\ AB: & 0 + w_0 = \Lambda(a^b, n) - F_{acc}(a^b[n]) \\ GDT: & 0 + w_0 = \Lambda(\delta g^e, n) \end{array}$$

3.4 Discretization of the static observation

Following the same procedure as in the previous section —discretize and arrange to $\ell + w = F(x)$— for each static observation equations, we obtain:

$$\begin{array}{ll} CUPT: & p_0 + w_p = r^e[n] \\ VUPT: & v_0 + w_v = v^e[n] \\ GUPT: & g_0 + w_g = \delta g^e[n] + \gamma(r^e[n]) \\ XOVER: & 0 + w_0 = \|\delta g^e[n] + \gamma(r^e[n])\| - \\ & -\|\delta g^e[k] + \gamma(r^e[k])\| \end{array}$$

3.5 Final INS/GPS-gravimetric network

As a result of the preceeding discussion, our problem can be reduced to the solution of the system of equations formed by the dynamic models —VEL, FB, WIB, AB, OB, GDT— and the static models —CUPT, GUPT, VUPT, XOVER. The above mathematical models have been implemented in the GeoTeX/ACX software system —Colomina et al. (1992)— developed at the ICC

Model	l	x
IMU data		
VEL	0	$r^e[n-1]$, $r^e[n+1]$, $v^e[n]$
FB	$f^b[n]$	$a^b[n]$, $q[n]$, $\delta g^e[n]$, $r^e[n]$ $v^e[n-1]$, $v^e[n]$, $v^e[n+1]$
WIB	$\omega^b_{ib}[n]$	$o^b[n]$, q[n-1], q[n], q[n+1]
OB	0	$o^b[n-1]$, $o^b[n]$, $o^b[n+1]$
AB	0	$a^b[n-1]$, $a^b[n]$, $a^b[n+1]$
GDT	0	$\delta g^e[n-1]$, $\delta g^e[n+1]$
GPS data		
CUPT	p_0	$r^e[n]$
VUPT	v_0	$v^e[n]$
Additional data		
GUPT	g_0	$\delta g^e[n]$, $r^e[n]$
XOVER	0	$r^e[n]$, $r^e[k]$, $\delta g^e[n]$, $\delta g^e[k]$

Table 1: INS/GPS-gravimetry mathematical models for NA.

since 1988. The associated observations and parameters for each model are shown in Table 1.

Solving the system is to perform an optimal estimation of its parameters in the sense of least-squares; i.e. the expectation of the parameters and their covariance is known. We emphasize the possibility to compute the covariance of a limited number of selected parameters —perform a selective inversion of the normal matrix— and the variance component estimation. In our case, these selected parameters will be the gravity disturbances and their covariances for further geoid determination.

4 Implementation issues

First consider that IMU data is collected at $Hz(IMU)$ frequency ($50Hz \leq Hz(IMU) \leq 200Hz$ for operational environments), the GPS data at $Hz(GPS)$ frequency ($1Hz \leq Hz(GPS) \leq 5Hz$) and the additional data can be collected a different frequencies. Second, consider that we want to compute all the parameters at $Hz(IMU)$ frequency. Then the size of the system increase considerably. This can be shown in Table 2 for some operational environments.

Although the large size of the associated normal matrices, they are essentially of the band-bordered type and we can apply sparse matrix techniques, fill-in reduction techniques and memory-to-disk paging to solve the system of linear equations. If this is done, the computational load is comparable to that of SSA.

	Case 1	Case 2	Case 3	Case4
$H_z(GPS)$	1	1	5	2
$H_z(IMU)$	200	50	200	50
t (s)	10 800	14 400	14 400	11 301
N_{XOVER}		30 000		332 150
N_{IMU}	2 160 000	720 000	2 880 000	565 050
N_{GPS}	10 800	14 400	72 000	22 602
N_{GUPT}				1 907
N^*_{eq}	38 912 400	13 033 200	52 056 000	10 572 763
N_{par}	38 880 000	12 960 000	51 840 000	10 170 900
r_b	0.0008	0.0056	0.0042	0.0380

Table 2: Information data of some operative environments at the ICC and UofC. Case 1: Applanix flight (ICC). Case 2: CASI flight (ICC). Case 3: GeoMobil (ICC). Case 4: Rockies'95 (UofC). $N_{eq} = N^*_{eq} + N_{al}$. $N^*_{eq} = 18N_{IMU} + 3N_{GPS}$. $r_b = \frac{N_{eq}-N_{par}}{N_{eq}}$ (average redundancy).

If it is still necessary to reduce the size of the system, it is possible to take into account that some of the parameters —or unknowns— have a slow variation in time and subsets of them can be grouped in a single one. This can considerably reduce the number of unknowns. For instance, it should be possible to compute a^b, o^b, δg^e at $Hz(GPS)$ frequency and r^e, v^e and q at $Hz(IMU)$ frequency. In this case, $Npar$ is reduced to half (19537200 in Case 1, 6609600 in Case 2, 26568000 in Case 3 and 5288868 in Case 4 respectively).

Table 2 also shows the small average redundancy of the systems to be solved ($0.3 \leq r_b \leq 0.9$ for geodetic and photogrammetric bundle networks). It is well-known that accurate least-squares adjustment needs high redundancy. Looking at Table 2, the number of IMU data and GPS data cannot be increased without increasing the number of parameters to be determined. Then to increase the redundacy of the system, we have to increase the number of additional equations. So, it is advisable to use all the existing observational information to increase the redundancy of the system.

It is under investigation the study of how to plan the surveys to increase them: cross-overs (repetition or/and intersection of flight lines), ZUPT periods at the beginning and at the end of the survey, upwarded gravity points, etc.

5 Conclusions, ongoing work and further research

It has been seen that the determination of the anomalous gravity by inertial techniques is critically employee of the capacity to separate the errors of the system of the effects of the gravitational field. This separation is based mainly on the different characteristics from both signals: the errors of the inertial sensors (INS) can be reasonably considered like time function, whereas the variations of the gravitational field are only function of the position.

Actually, SINS airborne gravimetry has been mainly based on Kalman filtering (SSA approach). In Kalman, the error separation is obtained, at first, by the use of different correlations of the bias and the variations of the gravitational field. The advantage of Kalman filter is the good physical description of the instruments errors, but it displays a serious disadvantage by the incapacity of handling space correlations, like the condition of cross-over points.

It has been presented that the development of an adjustment method in genuinely geodetic post-process with the explicit purpose to determine precise gravity anomalies taking advantage at maximum the space characteristics of the gravitational field. This method tries to jointly deal with the bias like function the time and the anomalous gravity like function of the position, by means of the resolution of the corresponding system of equations.

This system of equations can be, at first, very large and its redundancy small. It is under investigation some methods (numerical and geodetic) to handle with and to increase its redundancy.

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